Mathematical Analysis - List 2

1. Simplify each the following expressions in the given interval.

a) $x + |2 - x| + 3|1 - x|$, for $x \in (1, 2)$; b) $|2x| - |x + 1| + 2|x - 2|$, for $x \in (2, \infty)$;

c) $\dfrac{|x-1|}{|x+1|} - |2 - 3x|$, for $x \in (-\infty, -1)$;

2. Use the two properties of the absolute value: $|x| = |-x|$ and $|xy| = |x||y|$, and the fact that $|x - a|$ represents the distance between x and a to sketch (on the real line $\mathbb{R}$) the solution set for each of the following inequalities.

a) $|3x - 1| \leqslant 2$; b) $\dfrac{1}{2}|2 - x| < 1$; c) $|5 - 4x| > 3$; d) $|2 - 3x| \geqslant 4$.

3. Find the domain of each function.

a) $f(x) = \dfrac{x}{x^2 - 2x - 3}$; b) $f(x) = \dfrac{x-2}{x^2+4}$; c) $f(x) = \sqrt{16 - x^2}$;

d) $f(x) = \sqrt{-(x+3)^4}$; e) $f(x) = \dfrac{x-1}{\sqrt{x-1}}$; f) $f(x) = \dfrac{x-4}{x^2 - 8x + 16}$.

4. Find the range of each function.

a) $f(x) = x^2 + 2x$; b) $f(x) = -\sqrt{x} + 2$; c) $f(x) = \dfrac{x^2}{x^2+1}$; d) $f(x) = 1 + \dfrac{1}{x+1}$.

5. Show that the function $g(x) = \dfrac{3 + x^4}{x^2 + 2^{|x|}}$ is even and the function $f(x) = \dfrac{\sqrt{|x|}}{x^5 + x^3}$ is odd.

6. Determine whether f is increasing or decreasing on the given interval.

a) $f(x) = x^3 + 3x + 2$, $(-\infty, 0]$; b) $f(x) = -\sqrt{x-1}$, $[1, \infty)$;

c) $f(x) = \dfrac{1}{1+x^2}$, $[0, \infty)$; d) $f(x) = |x| - x$, $\mathbb{R}$.

7. A function f satisfies the following condition

$$\forall x \in \mathbb{R} \qquad f(x+1) = \frac{1 + f(x)}{1 - f(x)}.$$

Find $f(x + 2)$ and $f(x + 4)$, and deduce that f is periodic.

8. For each of the three "old" functions: $y = x^2$, $y = \dfrac{1}{x}$ and $y = |x|$ draw the graphs of the following "new" functions:

a) $y = x^2 - 2$, $y = -\dfrac{1}{2}x^2$, $y = (x+3)^2$, $y = x^2 - 4x + 7$;

b) $y = -\dfrac{1}{x}$, $y = \dfrac{2}{x}$, $y = \dfrac{1}{x+3}$, $y = 2 + \dfrac{3}{x-1}$, $y = \left|2 + \dfrac{3}{x-1}\right|$;

c) $y = |x - 2|$, $y = \dfrac{1}{3}|x|$, $y = 1 - |x|$, $y = |2x + 4| - 2$, $y = ||2x + 4| - 2|$.